Coastal Engineering Technical Note

SPECTRAL ANALYSIS OF DIGITAL WAVE DATA
COMPUTER PROGRAM: SPECTRUM

Purpose: This CETN introduces an interactive program for spectral analysis of water elevation data capable of fast execution on microcomputers.

Capabilities: Computer program SPECTRUM takes a time series of evenly spaced water elevation data and breaks it down into its surface variance spectral density. Under linear wave theory, the surface variance spectral density can be converted to spectral energy density upon multiplication by ρg - where ρ is the density of water and g is acceleration due to gravity. SPECTRUM is written in FORTRAN 77 and can handle a time series of up to 4096 points. SPECTRUM is very efficient in terms of memory usage and thus is attractive for use on microcomputers.

SPECTRUM will test for a trend in the time series and optionally delete it upon consent of the user. SPECTRUM also produces 90 percent confidence intervals for the actual surface variance spectral density values. SPECTRUM calculates the zeroth through fourth moments of the spectrum and several useful wave and spectral parameters including significant wave height and peak period. SPECTRUM also allows the user a choice of data windows or tapers. The interactive input has optional default settings for most of the input data.

Methods: The data analysis routine is performed using the Welch method (Welch, 1967) of spectral estimation which may be briefly summarized as follows:

(1) Segment the N point time series (Y_n) $n = 0,1,\ldots\ldots,N-1$ into K half-overlapping segments of L points each. This implies that

$$K = (N - L/2) / (L/2)$$ and we define the jth segment as

$$Z_{jn} = Y_{n+(j-1)\,L/2}$$ where

$$j = 1,2\ .\ .\ .\ .\ ,K$$

$$n = 0,1,\ .\ .\ .\ .\ ,L-1$$

(2) Subtract out the mean of each segment

$$X_{jn} = Z_{jn} - \overline{Z}_j \qquad \text{where}$$

$j = 1,2, \ldots, K$

$n = 0,1, \ldots, L-1$

$$\overline{Z}_j = \sum_{n=0}^{L-1} Z_{jn}/L$$

(3) Compute $G_j(m\Delta f)$, the discrete Fourier transform of the jth segment at frequency $m\Delta f$

$$G_j(m\Delta f) = \Delta t \sum_{n=0}^{L-1} X_{jn} W_n e^{-2\pi imn/L}$$

$j = 1,2 \ldots, K$

$m = 0,1, \ldots, L/2-1$

$i^2 = -1$

$\Delta f = 1/(N\Delta t)$

Δt = the recording time between consecutive data points

W_n = the data window for the nth point of the segment

(4) Compute the estimated surface variance spectral density from the jth segment $- \hat{S}_j(m\Delta f)$

$$\hat{S}_j(m\Delta f) = 2C_j \left| G_j(m\Delta f) \right|^2 / L\Delta t \qquad \text{where}$$

$j = 1,2, \ldots, K$

$m = 0,1, \ldots, L/2-1$

$|G_j(m\Delta f)|$ = the complex modulus (length) of $G_j(m\Delta f)$

$$C_j = L/\sum_{n=0}^{L-1} W_n^2$$

(5) Average the surface variance spectral density from each segment to obtain a final estimate which we shall denote by $\hat{P}(m\Delta f)$.

$$\hat{P}(m\Delta f) = \sum_{j=1}^{K} \hat{S}_j(m\Delta f)/K$$

The principle advantage of the Welch method in comparison to other non-overlapping segmenting techniques is that it reduces the width of the confidence interval for the actual surface variance spectral density values.

Additional details pertaining to the mathematics of spectrum and time series analysis may be found in Bendat and Piersol (1971), Borgman (1972), Borgman (1973), Lund (1986), and Welch (1967).

The discrete Fourier transform is performed using the fast Fourier transform algorithm which is an extremely efficient computational scheme for $G_j(m\Delta f)$. The fast Fourier transform routine in the program requires that the time series length N be a power of 2 - if not, the time series length is truncated to the largest power of 2 less than N. A 90 percent confidence interval for the actual surface variance spectral density value $P(m\Delta f)$ is calculated using the value of $\hat{P}(m\Delta f)$ and the chi-square probability law. The user may easily adjust SPECTRUM for percent values other than 90 percent by changing the upper tail $Z_{\alpha/2}$ value from the standard normal distribution at runtime. The zeroth through the fourth moments of the spectrum are calculated by the program. The mth moment, U_m, is defined as follows:

$$U_m = \int_0^\infty f^m \hat{P}(f)\, df \qquad m = 0,1,2, \ldots .$$

The spectral parameters are numbers which quantify the shape of the wave spectrum. They are usually functions of the moments. The spectral parameters calculated in SPECTRUM are listed and defined as follows:

The spectral width parameter $= \varepsilon = \sqrt{1 - (U_2{}^2/U_0U_4)}$

The spectral narrowness parameter $= \nu = \sqrt{(U_2U_0/U_1{}^2) - 1}$

The spectral peakedness parameter $= Qp = (2/U_0{}^2) \int_0^\infty f\, P(f)^2\, df$

The significant wave height $= H_s = 4\sqrt{U_0}$

These parameters are discussed in Rye (1979). Note that H_s by this definition is not necessarily equivalent to the average height of the one-third highest waves, particularly in shallow water. The integrals in the moments and equations defining the spectral parameters are calculated using the trapezoidal rule as the numerical integration scheme and $\hat{P}(m\Delta f)$ as an approximation for $P(m\Delta f)$.

The peak frequency is calculated by two different methods and is defined as the frequency at which the maximum surface variance spectral density occurs. The first method searches the array of variance density estimates for its maximum value; the second method averages all frequencies which have variance densities that exceed 0.6 times the peak frequency calculated by the first method. The peak period is computed as the reciprocal of the peak frequency computed in the first method.

The data windows are used in the reduction of side lobe leakage. This helps keep variance from "leaking" into frequencies where it does not belong. SPECTRUM gives the user a choice of three data windows:

1) The rectangular boxcar window

$$W_n = 1 \qquad 0 \leq n \leq L-1$$

2) The 10 percent cosine bell window

$$W_n = \begin{cases} 1/2[1 - \cos(10\pi n/L)] & 0 \leq n \leq L10 \\ 1 & L10+1 \leq n \leq L-L10-2 \\ 1/2[1 - \cos(10\pi(L-n-1)/L)] & L-L10-1 \leq n \leq L-1 \end{cases}$$

where L10 = [L/10] = the greatest integer less than or equal to L divided by 10.

3) The Hamming window

$$W_n = .54 - .46 \cos(2\pi n/L) \qquad 0 \leq n \leq L-1$$

Input: A title for each run is entered followed by the number of data points N, the segment length L, the data window choice, and the sampling frequency fs ($\Delta t = 1/fs$). Default values for the segment length and data window type are provided by the program. The default data window is the 10 percent cosine bell; while the default segment length is chosen such that the number of segments is seven. These values can be easily changed by the user at runtime. The sampling frequency can be interpreted as the number of data points recorded per second.

Next, SPECTRUM asks for lower and upper cutoffs for the computation of spectral moments and parameters (Rye, 1979). Cutoffs should be in terms of factors times the peak frequency. Default cutoffs are 0.3 and 3.0 times the peak frequency. Finally, the units of water elevation data and the name of the data file containing the water elevation data are input. The user should make sure that the format statement in the program reads the input time series in the correct format. This should be done by the user at runtime. The default format statement reads the data in a 6X,25F3.0 formatted data file. This means the data file will contain six blank spaces at the beginning of each row of data and will be followed by 25 three digit floating point numbers with no blanks between. If the data file is not in this format, the user will be allowed to enter the format appropriate for the specific data file. If a trend in the data is detected, the user is informed and has the option of deleting it or leaving it stand as is; if no trend is found, the program continues execution without interuption. All input time series should be reviewed and edited for quality before use with this program. Input data must be water level measurements. Erroneous results may be produced if this program is used to process pressure or current time series.

Output: The title of the run is printed as a heading for the program output. This is followed by a table of frequency verses surface variance spectral density. This table also includes 90 percent confidence intervals for the actual surface variance spectral density values. The confidence intervals may be interpreted as follows - actual surface variance spectral density falls between the lower and upper bounds with 90 percent confidence. The table is followed by output of the zeroth through fourth spectral moments. Next, the degrees of freedom for the chi-square distribution and the recording interval between consecutive data points (Δt) are printed. Then the spectral parameters are output. The significant wave height, spectral peakedness parameter, spectral width parameter, and spectral narrowness parameter are all displayed by the program. Finally, peak frequency and its reciprocal, peak period, are displayed by SPECTRUM.

Sample Problem: A twenty-minute time series from a Waverider buoy in Lake Michigan (South Haven, Michigan) is analyzed in the following sample execution of SPECTRUM.

```
B>SPECTRUM
***************************************************
*  SPECTRUM IS A PROGRAM WHICH BREAKS DOWN A TIME   *
*  SERIES OF WATER ELEVATION DATA INTO ITS SURFACE  *
*  VARIANCE DENSITY SPECTRUM.  SPECTRUM IS ABLE TO  *
*  CHECK FOR AND OPTIONALLY ELIMINATE TRENDS IN THE *
*  DATA.  SPECTRUM USES THE WELCH METHOD OF HALF-   *
*  OVERLAPPING SEGMENTS AND WILL PRODUCE CONFIDENCE *
*  INTERVALS FOR THE ACTUAL SPECTRAL VALUES.  THE   *
*  ZEROTH THROUGH THE FOURTH SPECTRAL MOMENTS AND   *
*  SEVERAL USEFUL SPECTRAL PARAMETERS ARE INCLUDED  *
*  IN THE PROGRAM OUTPUT.                           *
***************************************************

INPUT THE SITE OF DATA COLLECTION OR THE TITLE OF THIS RUN.
TRIAL RUN

HOW MANY DATA POINTS DO YOU HAVE IN YOUR TIME SERIES?
4096

DO YOU WISH TO ENTER A SEGMENT LENGTH (Y/N)?
IF NOT, ONE WILL BE SELECTED FOR YOU SUCH THAT
THE NUMBER OF SEGMENTS EQUALS 7.
N

DO YOU WISH TO ENTER A WINDOW TYPE (Y/N)?  IF NOT,
THE 10% COSINE BELL WINDOW WILL BE CHOSEN FOR YOU.
Y
RECTANGULAR BOXCAR WINDOW     = 1
10 PERCENT COSINE BELL WINDOW = 2
HAMMING WINDOW                = 3
SELECT 1, 2, OR 3
1

HOW OFTEN ARE THE DATA POINTS SAMPLED?
USE UNITS OF RECORDINGS PER SECOND (HZ).
4

DO YOU WISH TO ENTER HIGH AND LOW FREQUENCY CUTOFFS
FOR THE COMPUTATION OF THE SPECTRAL MOMENTS AND
PARAMETERS(Y/N)?
DEFAULT VALUES ARE 0.3 AND 3.0 TIMES THE PEAK FREQUENCY
N

WHAT ARE THE UNITS OF YOUR WATER ELEVATION DATA?
FEET...........1
INCHES.........2
CENTIMETERS....3
SELECT 1, 2, OR 3.
3

WHAT IS THE NAME OF YOUR DATA FILE?
BE SURE TO INCLUDE THE EXTENSION.

C:SHAV1720.DAT
```

THE DEFAULT FORMAT IS (6X,25F3.0)
IS YOUR DATA IN THIS FORMAT (Y OR N) Y

YOUR DATA INDICATES A LINEAR TREND WITH AT LEAST 95
PERCENT CONFIDENCE. DO YOU WANT TO REMOVE IT (Y/N)?
Y

TRIAL RUN

SPECTRAL DECOMPOSITION OF THE TIME SERIES:

FREQ(HZ)	SPECTRAL DENSITY CM^2 /HZ	90 PERCENT CONFIDENCE INTERVALS FOR THE ACTUAL SPECTRAL VALUES	
		LOWER	UPPER
.047	583.369	317.320	1501.989
.051	962.267	523.418	2477.529
.055	203.701	110.802	524.464
.059	51.610	28.073	132.879
.063	30.498	16.589	78.522
.066	43.365	23.588	111.650
.070	20.708	11.264	53.318
.074	81.608	44.390	210.115
.078	45.254	24.615	116.514
.082	25.424	13.829	65.459
.086	28.321	15.405	72.917
.090	36.365	19.781	93.629
.094	32.069	17.443	82.566
.098	152.352	82.871	392.257
.102	85.339	46.420	219.721
.105	84.036	45.711	216.365
.109	95.011	51.681	244.624
.113	158.233	86.070	407.398
.117	349.956	190.356	901.024
.121	480.614	261.426	1237.426
.125	1080.516	587.739	2781.982
.129	1840.780	1001.280	4739.419
.133	3501.738	1904.746	9015.853
.137	4975.678	2706.486	12810.780
.141	11350.140	6173.829	29222.960
.145	10564.840	5746.672	27201.080
.148	10258.390	5579.980	26412.070
.152	12410.590	6750.656	31953.300
.156	17176.920	9343.267	44225.060
.160	25040.200	13620.440	64470.480
.164	24908.960	13549.060	64132.600
.168	10246.520	5573.522	26381.500
.172	14238.730	7745.057	36660.160
.176	8955.002	4871.011	23056.260
.180	7385.563	4017.325	19015.460
.184	5700.385	3100.685	14676.660
.188	3712.231	2019.242	9557.804
.191	3798.051	2065.923	9778.763
.195	2936.927	1597.521	7561.646
.199	3033.121	1649.845	7809.314
.203	2535.227	1379.019	6527.396
.207	3630.544	1974.809	9347.486
.211	2704.000	1470.822	6961.933
.215	3080.744	1675.749	7931.927
.219	975.079	530.388	2510.517
.223	2411.549	1311.745	6208.966
.227	1527.485	830.865	3932.785
.230	1072.366	583.306	2760.999
.234	1679.959	913.802	4325.356
.238	2118.298	1152.233	5453.938
.242	1992.581	1083.851	5130.257
.246	2086.398	1134.882	5371.806
.250	791.320	430.433	2037.396
.254	638.298	347.198	1643.412
.258	2046.385	1113.117	5268.785
.262	1157.318	629.515	2979.722
.266	1480.093	805.086	3810.765
.270	1685.499	916.815	4339.619
.273	1039.320	565.331	2675.916
.277	619.954	337.220	1596.184
.281	1036.827	563.975	2669.496

.285	1399.931	761.482	3604.372
.289	1691.631	920.151	4355.409
.293	393.281	213.922	1012.572
.297	268.335	145.959	690.877
.301	723.807	393.710	1863.570
.305	513.601	279.370	1322.357
.309	774.767	421.429	1994.777
.313	388.513	211.329	1000.296
.316	541.822	294.720	1395.018
.320	430.128	233.965	1107.441
.324	598.140	325.354	1540.020
.328	301.869	164.200	777.216
.332	588.470	320.094	1515.120
.336	636.964	346.472	1639.978
.340	619.946	337.215	1596.162
.344	245.769	133.684	632.775
.348	412.251	224.241	1061.415
.352	399.952	217.551	1029.749
.355	340.992	185.480	877.945
.359	219.530	119.412	565.218
.363	402.318	218.838	1035.839
.367	524.848	285.487	1351.314
.371	410.288	223.173	1056.361
.375	200.606	109.118	516.495
.379	187.323	101.893	482.298
.383	168.245	91.516	433.177
.387	184.571	100.396	475.212
.391	137.873	74.995	354.978
.395	217.429	118.269	559.810
.398	228.819	124.465	589.136
.402	166.166	90.385	427.825
.406	110.602	60.161	284.764
.410	94.535	51.421	243.396
.414	102.065	55.517	262.784
.418	250.974	136.516	646.179
.422	157.331	85.579	405.078
.426	170.032	92.488	437.778
.430	131.411	71.480	338.342
.434	93.565	50.894	240.900
.438	115.813	62.996	298.181
.441	81.882	44.539	210.819
.445	90.164	49.044	232.144
.449	148.577	80.818	382.539
.453	129.928	70.674	334.523
.457	43.477	23.649	111.940
.461	97.676	53.130	251.484
.465	110.491	60.101	284.479
.469	154.909	84.262	398.841
.473	47.188	25.668	121.495
.477	45.413	24.702	116.924
.480	31.338	17.046	80.685

SPECTRAL MOMENTS ARE AS FOLLOW:

```
THE 0TH MOMENT OF THE SPECTRUM IS     934.347
THE 1TH MOMENT OF THE SPECTRUM IS     171.784
THE 2TH MOMENT OF THE SPECTRUM IS      34.466
THE 3TH MOMENT OF THE SPECTRUM IS       7.726
THE 4TH MOMENT OF THE SPECTRUM IS       1.968

THE DEGREES OF FREEDOM ARE                9.800
THE RECORDING INTERVAL IS                  .250 SECONDS
SIGNIFICANT WAVE HEIGHT IS              122.268 CM
SPECTRAL PEAKEDNESS PARAMETER IS          1.965
THE SPECTRAL WIDTH PARAMETER IS            .595
SPECTRAL NARROWNESS PARAMETER IS           .302
THE PEAK FREQUENCY IS                      .160 HZ
THE DELFT PEAK FREQUENCY IS                .160 HZ
THE PEAK PERIOD IS                        6.244 SECONDS

Stop - Program terminated.
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Program Availability: SPECTRUM is available in Microsoft FORTRAN for the IBM-PC on a 5-1/4 in. diskette or as a printed program listing and may be obtained from Ms. Gloria J. Naylor at (601) 634-2581 , Engineering Computer Programs Library Section, U.S. Army Engineer Waterways Experiment Station, P.O. Box 631, Vicksburg, MS 39180-0631. Questions concerning applications of SPECTRUM can be directed to (601) 634-2012 the Coastal Design Branch, Coastal Engineering Research Center, U.S. Army Engineer Waterways Experiment Station, P.O. Box 631, Vicksburg, MS 39180-0631.

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